Topological approach to Short-Term Synaptic Plasticity in Neurons.
A Review.

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Abstract
The experimental work of many groups over the years shows that many synapses exhibit short-term plasticity. Here, the emphasis is on the phrase short-term as opposed to long-term changes that are associated with learning and memory. Short-term plasticity occurs over timescales of the order of milliseconds to minutes and takes the form of short-term depression (the magnitude of successive Post Synaptic currents decreases), facilitation (the magnitude of successive Post Synaptic currents increases), or possibly both. Dynamical system approach, has been used to study plasticity, but lately a new approach to Neuroscience is coming from Topology and in particular Algebraic Topology seems could be a fresh point of view to study the short-term plasticity in neurons.

Introduction

There are many other channels on the surface of nerve cells, besides the voltage- and ion-gated channels, which respond to various substances. Indeed, we need to introduce an additional class of membrane channels. Among the most important of these, at least in computational neuroscience, are synaptic channels. It is well known that the transmitters associated with cortical neurons are Glutamate and γ-aminobutyric acid (GABA). The Glutamate excites the postsynaptic cell, whereas GABA inhibits it. However, the main problem of some GABA receptors is that its reversal potential is highly dependent on chloride concentration, so it cane close to rest and even above rest. Thus, (particularly, early in development) some GABA synapses can be excitatory.Transmitter released can become quite complex and the release of transmitter is probabilistic and occurs in discrete amounts called *quanta*. Presynaptic stimulation, leads into the next presynaptic spike, more transmitter is released than on the first spike. This increase is called *Potentiation* or *Facilitation*. Additionally, after several presynaptic spikes, the transmitter release per spike can decrease through various means (such as depletion) and take some time to recover. Decrease of transmitter over successive firings of action potentials is called synaptic *Depression*. Thus, in this way the *Short-Term plasticity* e.g., depression and facilitation) is generated. On the other hand, *Long-Term Plasticity* has to do with memory. One of the main hypotheses in neuroscience is that memories are encoded in the strengths of synapses between neurons.

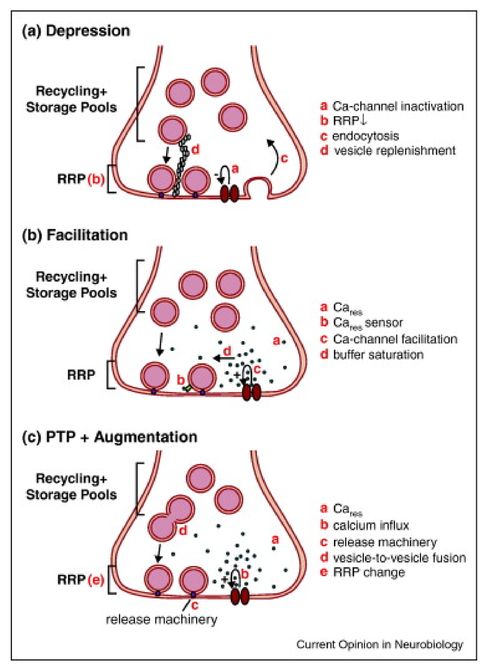

Figure 1: Short-term Plasticity [11]

Main Objectives

The main goal of this study are:

1. To understand the concept of short-term plasticity and compare it with long-term plasticity.
2. To learn the classical approach using Dynamical systems.
3. To introduce the foundational assumptions, central ideas, and dominant criticisms from Algebraic Topology that can be applied to understand better the short-term plasticity.

Dayan and Abbott Model [6]

Suppose we want to characterize the *magnitude,* $M(t)$, *of synaptic release per presynaptic spike*. We write this magnitude as the product of two factors, the depression $q(t)$ and the facilitation $f(t)$, so that

$$M(t) = q(t)f(t) \tag{1}$$

Both $f(t)$ and $q(t)$ lie between 0 and 1 and each has a resting value of f_0 and d_0, respectively, to which it returns with time constant τ_f and τ_d, respectively.Thus, in absence of any inputs,

$$\tau_f \frac{df}{dt} = f_0 - f \quad and \quad \tau_d \frac{dq}{dt} = d_0 - q \tag{2}$$

Each time there is a spike, $f(t)$ is increased by an amount $a_f(1-f)$ and $q(t)$ is decreased by an amount $a_d q$. Formally, we can write the facilitation equation as:

$$\frac{df}{dt} = \frac{f_0 - f}{\tau_f} + \left(\sum_j \delta(t-t_j)\right) a_f(1-f) \tag{3}$$

y similarly for the depression equation

$$\frac{dq}{dt} = \frac{d_0 - q}{\tau_d} + \left(\sum_j \delta(t-t_j)\right) a_d q \tag{4}$$

Now, let's suppose the inputs to the synapse are *Poisson* with rate r and averaging the first equation

$$\frac{df}{dt} = \frac{f_0 - f}{\tau_f} + a_f r(1-f) \tag{5}$$

from the above equations we have the steady-state values of f and q are:

$$f_{ss} = \frac{f_0 + a_f\tau_f r}{1 + a_f\tau_f r} \qquad q_{ss} = \frac{d_0}{1 + a_d\tau_d r} \tag{6}$$

The effective average rate is

$$r_{eff} = r f_{ss} q_{ss} = r d_0 \frac{f_0 + a_f\tau_f r}{(1 + a_f\tau_f r)(1 + a_d\tau_d r)} \tag{7}$$

If there is depression, then this function saturates as the true rate goes to infinity.

Algebraic Topology

The structure of white matter - *the connectome* - is essentially the brains wiring diagram (it transmits information between different parts of the brain), and it is poorly understood. One way to investigate this structue is thanks to the mathematical field of Algebraic Topology, which neurologists are gradually coming to grips with for the first time. With this idea in mind, Algebraic Topology is used to set up the challenging goal of finding symmetries in topological spaces at different scales, which could be a key in understanding the connectome.

Figure 2: The Connectome [8]

The Neural Space
In their pioneering work D. Hubel and T. Wiesel [[9], [10]] demonstrated that visual information is encoded in the visual cortex of the brain. Thus, it seems that our perception of space is connected to the brain certain areas and certain activities. o a given coherent collection of neurons like place cells and grid cells it is reasonable to suggest that the naturally associated term *Neural Space* is used tentatively for the set of spike trains with a correlation "distance".

Simplicial Complexes
It seems [7] that the topological theory of simplicial complexes provides a remarkably efficient semantics for describing many familiar concepts and phenomena of the brain (hippocampal) physiology. Thus, it is possible to build different sets ad-hoc for this study, namely, *A nerve complex* $\mathcal{N}$; *The coactivity complex* $\mathcal{T}$; *Cell assembly complex* $\mathcal{T}_{\mathcal{CA}}$; and together with this, *A simplicial schema of a memory space* through a finite topological space.

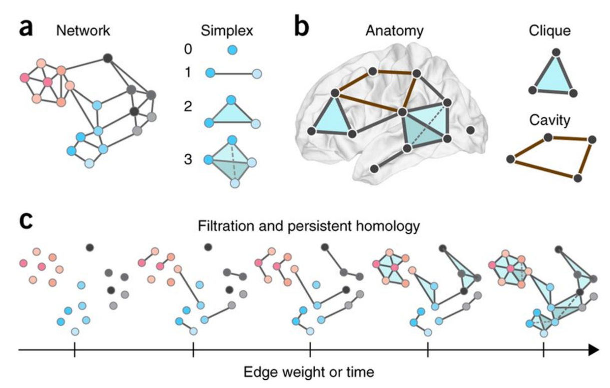

Figure 3: Simplicial Complex [5]

Cohomology and Betti-Curves
This ideas were introduced by Curto, Itskov, Guisti, and Dabaghian, in a serie of papers. Here the concept that is used is *persisten cohomology*, where they convert the train of spikes into persistence diagrams or *Betti-curves* . This is done by taking in account that a given family of spike trains from a group of neurons, then we introduce a metric there ("correlation distance") of that train. From this set of data they construct a filtered space and compute the topological holes measured by homology theory.
Short-Term and Long-Term Synaptic plasticity Is through this model that we could give a fresh point of view of the behavior of the brain, in the sense of functional plasticity. For instance, the main mechanism of Plasticity, *Facilitation* and *Depression*, they can be cataloged and understood through betti-curves or another topological properties of the Simplicial Complexes.

Conclusions

- This project even though is in its early stages, shows that has a good future.
- It is clear that the Mathematical Neuroscience Community is having a strong tool in Algebraic Topology.
- In particular the understanding of Plasticity (Short-Long term) will find a good niche in the concept of Algebraic Topology.

Forthcoming Research

- Make a deeper review of what has been done in this line.
- Understand, through some models, that Plasticity (Short-Long term) will find a good niche in the concepts of Algebraic Topology.
- In particular understand Short-Term plasticity through persistent cohomology and simplicial complexes.

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